PT-003-1164003 Seat No. ________

M. Sc. (Sem. IV) Examination

August - 2020

Mathematics : CMT - 4003

(Number Theory - 2)

Faculty Code : 003
Subject Code : 1164003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) Attempt all the questions.
(3) Each question carries equal marks.

1 Answer any **seven** of the following : **14**

(1) Find the value of r_0, r_1, r_2 and r_3 for the continued fraction expansion $<1,1,1,1,2>$.

(2) Find the general solution of Diophantine equation $4x+8y=42$.

(3) Find two Primitive Pythagorean triplet (x, y, z) for which $z<59$.

(4) Define with examples : (a) Quadratic irrational and (b) Pell's Equation.

(5) Find two positive integers n such that $1+2+3+\cdots+n$ is a perfect square.

(6) Express the rational numbers $\frac{101}{7}$ and $\frac{1437}{11}$ as simple contiued fractions.

(7) Find the continued expansion of $\frac{\sqrt{5}+1}{2}$ and $\frac{\sqrt{5}-1}{2}$.

(8) Show that $\gcd(x, y) = \gcd(y, z)$, where (x, y, z) is a Pythagorean Triplet.

(9) Write down the Farey fractions between 0 and 1 in the rows upto 7^{th} row.

(10) Find the value of $<1,1,2,2,2,.......>$ and $<-2,2,4,3,3>$.

2 Answer any **two** of the followigng : **14**

(1) State and prove Hurwitz Inequality for simple continued fractions. **7**

(2) (a) Prove that for an irrational number x the infinite continued expansion is always unique. **4**

(b) If $a_0 a_1 a_2 \ldots\ldots, a_n, \ldots\ldots$ is sequence of integers with $a_i \geq 1$; for $i = 1, 2, 3, \ldots.,$ then show that $|xk_n - h_n| < \left|\frac{1}{k_{n+1}}\right|; \forall n,$ where x is in irrational number. **3**

(3) Suppose $u \neq 0$ is an integer and $v > 1$ is not a perfect square with $|u| < \sqrt{v}$. If (a, b) is the positive solution of $x^2 - vy^2 = u$ then $a = h_n$ *and* $b = k_n$, for some n provided $(a, b) = 1$. **7**

3 Answer the following : **14**

(1) (a) Find three positive solutions of the equation $x^2 - 3y^2 = 1$. **3**

(b) Justify : Is $x^2 - 15y^2 = -1$ has a solution in integers ? **4**

(2) Suppose $< a_0, a_1 a_2 \ldots\ldots, a_n, \ldots >$ be an infinite sequence of integers with $a_i \geq 1$; for $i = 1, 2, 3, \ldots\ldots n$ then prove that the subsequences r_{2j} and r_{2j-1} both converge to the same point, where h_j, k_j and r_j are defined as usual. **7**

OR

3 Answer the following : **14**

(1) Prove that if θ is an irrational number and suppose for some rational number $\frac{a}{b}$ for $b > 0$ and $(a, b) = 1$ with $|\theta b - a| < |\theta k_n - h_n|$ for some n then $b \geq k_{n+1}$. **7**

(2) (a) Prove that the equation $30x^2 - 14y^2 = 18$ does not have solutions in integer. **3**

(b) Find four solution of the equation $x^2 - 29y^2 = 1$. **4**

4 Answer the following : **14**

(1) Write an algorithm to find the sequence of integers $a_0, a_1 a_2 \ldots\ldots, a_n, \ldots\ldots$ when an irrational number is given and then explain with an example. **7**

(2) Prove that the value of $f(x) = x^4 + x^3 + x^2 + x + 1$ is a perfect square only for $x = -1, 0, 3$ otherwise $f(x)$ is not a perfect square. **7**

5 Answer any **two** of the following : **14**

(1) If $x > 1$ and $-1 < x' < 0$ the show that the continued fraction expansion of x is purely periodic provided x is a quadratic irrational. **7**

(2) Show that if the triplet (x, y, z) is a Primitive Pythagorean triplet then there exists r and s such that $r > s \geq 1.(r, s) = 1$ and r is even then s is odd and vice–versa. **7**

(3) Show that the equation $x^2 = y^3 + 7$ has no solution in integers. **7**